Calculus II – Pieces of Eight

Directions: Arrange these pieces into exactly EIGHT groups of equal or similar sequences. Assume c, k, p, a_i, and k_i are real numbers. Assume n represents only natural numbers.

<table>
<tr><td>$-1, 1, -1, 1, -1, 1, \ldots$</td><td>$\left\{\dfrac{\cos(n\pi)}{n}\right\}_{n=1}^{\infty}$</td><td>$\left\{\cos(n\pi)\right\}_{n=1}^{\infty}$</td></tr>
<tr><td>$6, 3, 1.5, 0.75, 0.375, \ldots$</td><td colspan="2" rowspan="2">$a_n = \dfrac{1}{\sqrt{p_n}}$, where $n \ge 1$
and p_n is the n^{th} prime</td></tr>
<tr><td>$3, -6, 12, -24, 48, \ldots$</td></tr>
<tr><td>$1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$</td><td colspan="2">$a_n = a_{n-1} + c$ for $n > 1$, $a_1 = k$</td></tr>
<tr><td>$1, 1, 2, 3, 5, 8, 13, 21, \ldots$</td><td colspan="2">$a_n = -2 \cdot a_{n-1}$ for $n > 1$, $a_1 = k$</td></tr>
<tr><td>$0, \frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25}, \ldots$</td><td colspan="2">$a_n = a_1 \cdot r^{n-1}$ for $n > 1$, $a_1 = k$</td></tr>
<tr><td>$20, 13, 6, -1, -8, -15, \ldots$</td><td colspan="2">$a_n = r \cdot a_{n-1}$ for $n > 1$, $a_1 = k$</td></tr>
<tr><td>$a_n = a_{n-1} + a_{n-2}$ for $n > 2$,
$a_1 = k_1$, $a_2 = k_2$</td><td colspan="2">$a_n = -1 \cdot a_{n-1}$ for $n > 1$, $a_1 = k$</td></tr>
<tr><td>$-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}, \ldots$</td><td colspan="2">$1, \frac{1}{8}, \frac{1}{27}, \frac{1}{64}, \frac{1}{125}, \ldots$</td></tr>
<tr><td>$1, 4, 7, 10, 13, 16, 19, \ldots$</td><td colspan="2">$a_1, \left\{a_1 + (n-1) \cdot c\right\}_{n=2}^{\infty}$</td></tr>
<tr><td>$a_n = (-1)^n$ for $n \ge 1$</td><td colspan="2">$1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{2}, \frac{1}{\sqrt{5}}, \frac{1}{\sqrt{6}}, \ldots$</td></tr>
<tr><td>$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$</td><td colspan="2">$a_n = \dfrac{n-1}{n^2}$, $n \ge 1$</td></tr>
<tr><td>$a_n = \dfrac{(-1)^n}{n}$, $n \ge 1$</td><td colspan="2">$\left\{\dfrac{1}{n^p}\right\}_{n=1}^{\infty}$ and $p > 0$</td></tr>
</table>

Created by Gary Nonnemacher, Muskegon Community College

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Solution

A = Arithmetic (4 pieces)

$20, 13, 6, -1, -8, -15, \ldots$

$1, 4, 7, 10, 13, 16, 19, \ldots$

$a_n = a_{n-1} + c \text{ for } n > 1,\ a_1 = k$

$a_1, \{a_1 + (n-1)\cdot c\}_{n=2}^{\infty}$

B = Alternating (4 pieces)

$-1, 1, -1, 1, -1, 1, \ldots$

$\{\cos(n\pi)\}_{n=1}^{\infty}$

$a_n = -1\cdot a_{n-1} \text{ for } n > 1,\ a_1 = k$

$a_n = (-1)^n \text{ for } n \geq 1$

C = Alternating Harmonic (3 pieces)

$-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}, \ldots$

$\left\{\frac{\cos(n\pi)}{n}\right\}_{n=1}^{\infty}$

$a_n = \frac{(-1)^n}{n},\ n \geq 1$

G = Geometric (5 pieces)

$6, 3, 1.5, 0.75, 0.375, \ldots$

$3, -6, 12, -24, 48, \ldots$

$a_n = -2\cdot a_{n-1} \text{ for } n > 1,\ a_1 = k$

$a_n = a_1 \cdot r^{n-1} \text{ for } n > 1,\ a_1 = k$

$a_n = r \cdot a_{n-1} \text{ for } n > 1,\ a_1 = k$

P = Power (5 pieces)

$1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

$1, \frac{1}{8}, \frac{1}{27}, \frac{1}{64}, \frac{1}{125}, \ldots$

$1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{2}, \frac{1}{\sqrt{5}}, \frac{1}{\sqrt{6}}, \ldots$

$\left\{\frac{1}{n^p}\right\}_{n=1}^{\infty} \text{ and } p > 0$

Q = Quotient (2 pieces)

$0, \frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25}, \ldots$

$a_n = \frac{n-1}{n^2},\ n \geq 1$

F = Fibonacci (2 pieces)

$1, 1, 2, 3, 5, 8, 13, 21, \ldots$

$a_n = a_{n-1} + a_{n-2} \text{ for } n > 2,\ a_1 = k_1,\ a_2 = k_2$

R = ARRGH! (arrgh … like a pirate, get it?)

$a_n = \frac{1}{\sqrt{p_n}}, \text{ where } n \geq 1 \text{ and } p_n \text{ is the } n^{\text{th}} \text{ prime}$

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