Road to U-Substitution Game (Calculus)

Learning goal: To begin to "see" the u and du substitutions in problems and to recognize compositions for what they are. I recommend introducing the game in two phases, because it's difficult for students to see how to structure u-substitution problems at first.

This is (ideally) a 3-player game, although it can be modified for two people.

Each group of players will need a game board, a set of player road pieces (the green, yellow, and blue pieces),three Integral Construction Platforms (one for each player) and a deck of cards (rectangular grey or white cards). Print all game pieces on cardstock for best results.

Phase I: Guided Play

Give each set of player a game board and an Integral construction board and that is all. On a document camera, lay out 8 cards so that everyone in the class can see them and then demonstrate how to create one of the integrals on the game board. To create an integral, you need a u, a du, and some function or combination of functions to put them in (unless, of course, you're using the integral of udu).

Example: The following could be used to construct $\int \left(x^2+2\right)^3 2x\,dx = \int u^3 du$.

$(\quad)^3$	x^2+3	$2x$

Example: The following four tiles could be used to construct $\int \frac{\sec^2 x}{(\tan x)^2} dx = \int \frac{du}{u^2}$.

$(\quad)^2$	$\frac{(\quad)}{(\quad)}$	$\tan x$	$\sec^2 x$

Walk the learners through one or two examples using the document camera. Every time you construct an integral, place a road piece on the game board on one of the edges of the appropriate hexagon, and then replace the missing cards.

Now, lay out a new set of cards for the **learners** to try constructing integrals (make sure to lay out 8 cards that have possibilities). Go through a couple rounds, with the "game" being the construction of as many integrals as possible OR using as many cards at once as you can in one turn.

Now the students are ready to play on their own! Let the real game begin!

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Phase II: Group Play

To Win: Build the longest continuous road on the board. With each integral you construct, you have the opportunity to build a single piece of the road on the edge of the appropriate hexagon.

Rules:

1. Every player is dealt 8 cards. Lay them out so that one row of 4 is above the Integral Construction Platform and one row of 4 is below the Integral Construction Platform. Anything that is above the card is fair game to be stolen from another player (more on this later), so keep your most valuable cards below or on the construction platform. You must always keep 8 cards out and 4 cards above the platform. No more, no less. As cards are played, they should be replaced from the deck.
2. On your turn, you can do two things. **Option 1:** Ideally, you construct an integral using a selection of game pieces, and place a road piece on the board. I think it is best if students write out the integral they are constructing on a sheet of scratch paper to present to their opponents (otherwise it's just a little too easy to let one slide by). **Option 2:** If you can't construct any integrals, you will have to sacrifice two cards to the other two players (one to each). What they take is their choice, first come first served (but they can only choose from the cards above the Construction. You can then replace these two missing cards from the deck, and each of the other players must get rid of a card (they may discard the one they just stole from you if they wish).
3. When a hexagon on the board has been surrounded by roads, you can remove all grey function cards that apply only to that hexagon from the game.

The game ends when the board is filled or when there is an indisputable winner no matter what else happens.

Two-player modification: If a player can't construct any integrals, then the opposing player steals two cards (instead of one).

Other modifications: Cut out the hexagons and place them in some other arrangement (I'd suggest making a copy to create a new fixed gameboard or there are too many moving pieces).

$e^{(\quad)}$	$e^{(\quad)}$	$e^{(\quad)}$	x^3	$3x^2$
$(\quad)^2$	$(\quad)^2$	$(\quad)^2$	$(\quad)^2$	$(\quad)^2$
$\sec^2(\quad)$	$\sec^2(\quad)$	$\sec^2(\quad)$	x^2	$2x$
$(\quad)^3$	$(\quad)^3$	$(\quad)^3$	x^2+3	$2x$
$\dfrac{(\quad)}{(\quad)}$	$\dfrac{(\quad)}{(\quad)}$	$\dfrac{(\quad)}{(\quad)}$	$\dfrac{(\quad)}{(\quad)}$	$\dfrac{(\quad)}{(\quad)}$

$\sqrt{(\quad)}$	$\sqrt{(\quad)}$	$\sqrt{(\quad)}$	$\tan x$	$\sec^2 x$
$\sin(\quad)$	$\sin(\quad)$	$\sin(\quad)$	$\sin x$	$\cos x$
$\cos(\quad)$	$\cos(\quad)$	$\cos(\quad)$	$\cos x$	$-\sin x$
e^{3x}	2^x	$\sec x$	e^x	$\ln x$
$3e^{3x}$	$2^x \ln 2$	$\sec x \tan x$	e^x	x^{-1}

x^3+1	$6x$	$6x$	x^3	$3x^2$
$\tan x$	$\sin x$	$\cos x$	x^2+3	x^2
$\sec^2 x$	$\cos x$	$-\sin x$	$2x$	$2x$
e^{3x}	2^x	$\sec x$	e^x	$\ln x$
$3e^{3x}$	$2^x \ln 2$	$\sec x \tan x$	e^x	x^{-1}

Integral Construction Platforms (one for each player)

$\int$	dx
$\int$	dx
$\int$	dx

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Road to U-substitution Game Board

$\int u\,du$

$\int \frac{du}{u^2}$

$\int u^3\,du$

$\int \frac{du}{u}$

$\int \cos u\,du$

$\int e^u\,du$

$\int u^{1/2}\,du$

$\int \sec^2 u\,du$

$\int u^2\,du$

$\int \sin u\,du$

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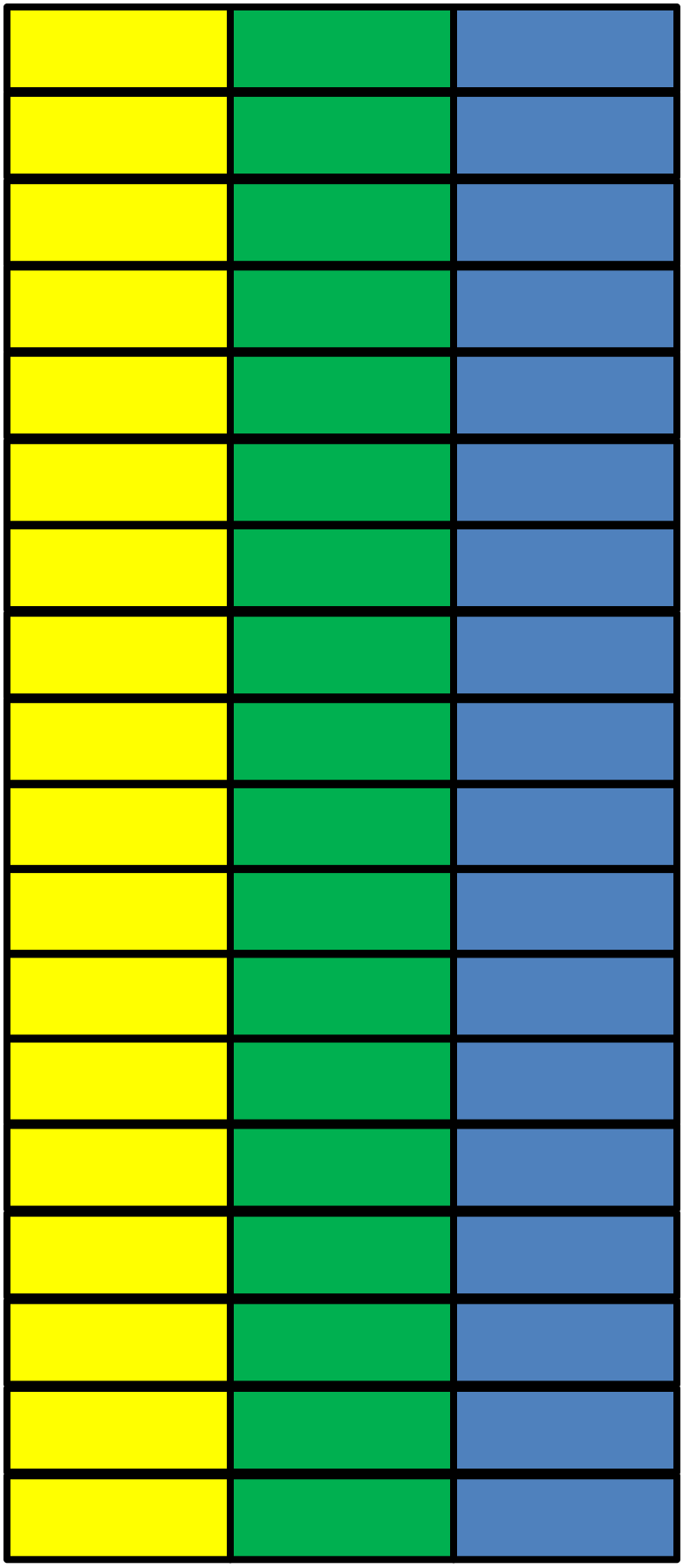
Player Road Pieces
(each player is a different color of road)



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